1. (a) Discuss the convergence of the series:

(i) \( \sum (\sqrt{n^4 + 1} - \sqrt{n^4 - 1}) \)

(ii) \( x \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \ldots \)

(b) If \( \alpha, \beta \) are the roots of \( x^2 - 2x + 4 = 0 \), prove that

\[
\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}.
\]

2. (a) Obtain the eigen values and eigen vectors for the matrix:

P. T. O.
\[
\begin{bmatrix}
2 & 0 & 1 \\
0 & 2 & 0 \\
1 & 0 & 2
\end{bmatrix}.
\]

Verify that the eigen vectors are orthogonal.

(b) If \( y^{1+m} + y^{-1+m} = 2x \), prove that:
\[
(x^2 - 1)y_{n+2} + (2n + 1)x y_{n+1} + (n^2 - m^2)y_n = 0.
\]

3. (a) Calculate the approximate value of \( \sqrt{10} \) to four places of decimal by taking the first four terms of an appropriate expansion.

(b) Find the half-range cosine and sine series for the function:
\[
f(x) = x(\pi - x), \quad 0 \leq x \leq \pi.
\]

4. Solve:

(a) \[
\frac{d^3y}{dx^3} + 2 \frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x
\]

(b) \[
\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = x \sin x
\]

5. (a) Evaluate \( \int \int_A xy \, dx \, dy \), where \( A \) is the domain bounded by x-axis, ordinate \( x = 2a \), and the curve \( x^2 = 4ay \).
(b) Evaluate:
\[ \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} \, dz \, dy \, dx. \]

6. (a) Trace the curve:
\[ 3ay^2 = x(x-a)^2 \]

(b) Find the area of ellipse
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

7. (a) Show that for the cardioid
\[ r = a(1 + \cos \theta), \]
\[ \frac{\theta^2}{r} \] is constant.

(b) Find the asymptotes of the curve:
\[ x^3 + 4x^2y + 5xy^2 + 2y^3 + 2x^2 + 4xy + 2y^2 - x - 9y + 2 = 0 \]