B.Tech. (C) / I
A

PAPER ECE-101— MATHEMATICS – I

Time: 3 hours

Maximum Marks: 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions.
All questions carry equal marks.

Assume missing data suitably, if any.

1. (a) Find the Laplace transform of \( \sin \omega t \) and \( \cos \omega t \). 7

(b) Find the inverse Laplace transform of \( \frac{s}{s^4 + s^2 + 1} \). 7

2. (a) Discuss the convergence of the series:
\[
\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \ldots \quad (0 < x < 1).
\]
7

(b) Evaluate \( \int \int_R xy(x+y) \, dx \, dy \), where R is the region bounded by \( y = x^2 \) and \( y = x \). 7

3. (a) Find the points of maxima and minima of \( x^3 y^2(1-x-y) \). 7

P. T. O.
(b) Explain in detail the physical meaning of the divergence and show that the vector field defined by $\vec{F}=e^{x+y-2z}(i+j+k)$ is solenoidal.

4. Solve the following differential equations:

(a) $y^{IV}+3y^{II}=108e^x$

\[
\begin{align*}
\frac{dy_1}{dt} + 2\frac{dy_2}{dt} - 2y_1 - y_2 &= e^{2t} \\
\frac{dy_2}{dt} + y_1 - 2y_2 &= 0
\end{align*}
\]

(b) \[
\begin{align*}
y_1'(t) + y_2'(t) + x(t) &= -e^{-t} \\
x'(t) + 2y'(t) + 2x(t) + 2y(t) &= 0
\end{align*}
\]

with $x(0) = -1$ and $y(0) = 1$

5. (a) Find the Fourier expansion of the function

$f(x) = x^2$, $-2 \leq x \leq 2$

(b) Find the Fourier expansion for $f(x)$ given by:

$$f(x) = \begin{cases} 
\pi + x, & -\pi < x < 0 \\
0, & 0 \leq x < \pi.
\end{cases}$$

6. (a) Using Laplace Transforms, solve the following system of differential equations:

\[
\begin{align*}
x'(t) + y'(t) + x(t) &= -e^{-t} \\
x'(t) + 2y'(t) + 2x(t) + 2y(t) &= 0
\end{align*}
\]

(b) Find the value of $\lambda$ for which the following equations possess a non-trivial solution and also find the corresponding solutions.
(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0
(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0
2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0

7. (a) Using elementary transformation, find the rank of the matrix
\[
\begin{bmatrix}
2 & 3 & 1 & 0 & 4 \\
3 & 1 & 2 & -1 & 1 \\
4 & -1 & 3 & -2 & -2 \\
5 & 4 & 3 & -1 & 5
\end{bmatrix}
\]

(b) State De Moivre’s theorem and use the same to solve the equation
\[x^4 - x^3 + x^2 - x + 1 = 0.\]

8. (a) Find the inverse of the matrix A, if it exists:
\[
A = \begin{bmatrix}
1 & 3 & 4 \\
3 & -1 & 6 \\
-1 & 5 & 1
\end{bmatrix}
\]

(b) Prove that the sum of the eigenvalues of a matrix A is the sum of the elements of its principal diagonal.