1. (a) Discuss the nature of the following series:

(i) \( \frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \ldots \)

(ii) \( 1 - \frac{x}{1^2} + \frac{x^2}{2^2} - \frac{x^3}{3^2} + \ldots \)

(b) If \( x_r = \cos \left( \frac{\pi}{2^r} \right) + i \sin \left( \frac{\pi}{2^r} \right) \), show that

\[ x_1 \cdot x_2 \cdot x_3 \ldots \infty = -1. \]

2. (a) Determine the value of \( \lambda \) for which the system of equations:

P. T. O.
\[ x_1 + x_2 + x_3 = 2 \]
\[ x_1 + 2x_2 + x_3 = -2 \]
\[ x_1 + x_2 + (\lambda - 5)x_3 = \lambda \]

(i) has no solution

(ii) has a unique solution.

(b) Find the eigenvalues and eigenvectors of the matrix:

\[
A = \begin{bmatrix}
8 & -6 & 2 \\
-6 & 7 & -4 \\
2 & -4 & 3
\end{bmatrix}
\]

3. (a) Find the asymptotes of the curve \( x^3 + 4x^2y + 5xy^2 + 2y^3 + 2x^2 + 4xy + 2y^2 - x - 9y + 2 = 0 \).

(b) Find the Fourier series to represent \( x \sin x \) from \( x = -\pi \) to \( x = \pi \).

4. Solve the following differential equations:

(a) \((D^2 - 1)y = \cosh x \cos x\)

(b) \[
\begin{cases}
\frac{d^2x}{dt^2} - 3x - 4y = 0 \\
\frac{d^2y}{dt^2} + x + y = 0
\end{cases}
\]
5. (a) Evaluate:
\[ \int_{0}^{1} \left( \int_{0}^{\sqrt{1-x^2}} \left( \int_{0}^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} \, dz \right) \, dy \right) \, dx. \]

(b) Evaluate the following integral by changing the order of integration:
\[ \int_{0}^{\infty} \left( \int_{x}^{\infty} \frac{e^{-y}}{y} \, dy \right) \, dx. \]

6. (a) For the curve \( r^2 = a^2 \cos 2\theta \), show that:
\[ \rho = \frac{a^2}{3r}. \]

(b) Show that the length of the curve \( y = \log \sec x \) between the points \( x = 0 \) and \( x = \pi/3 \) is \( \log (2 + \sqrt{3}) \).

7. (a) Given \( \sin 30^\circ = \frac{1}{2} \), use Taylor's Theorem to evaluate \( \sin 31^\circ \) correct to four significant digits given that \( \cos 30^\circ = 0.8660 \).

(b) If \( x + y = 1 \), prove that:
\[ D^n(x^n y^n) = n! \left[ y^n - \binom{n}{1} \left( \binom{n}{1} \right)^2 y^{n-1} x \right. \]
\[ \left. + \binom{n}{2} \left( \binom{n}{2} \right)^2 y^{n-2} x^2 - \ldots + (-1)^n x^n \right]. \]