B.Tech (C) / I

ECE - 101
Paper - MATHEMATICS - I

Time : 3 hours

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions. Assume missing data, if any. All questions carry equal marks.

1 a) Find Fourier series for the function \( f(x) \) given by
\[
    f(x) = \begin{cases} 
        1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 
        1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi 
    \end{cases}
\]

b) Find half range cosine series for \( f(x) = 2x - 1 \) in the interval \( 0 < x < 1 \). Hence show that
\[
    \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \]

2 a) Find the critical points for the function
\( f(x, y) = x^3 + y^3 - 3xy + 1 \) and test their nature.

b) Change the order of integration and evaluate
\[
    \int_0^2 \int_{x^2}^1 (x^2 + y^2) \, dy \, dx
\]

P.T.O
3  a) Find the eigen values and eigen vectors of the matrix

\[
A = \begin{bmatrix}
1 & 0 & -1 \\
1 & 2 & 1 \\
2 & 2 & 3 \\
\end{bmatrix}
\]

b) If \( \cosh(u + iv) = x + iy \), prove that

\[
\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1 \quad \text{and} \quad \frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1
\]

4  Solve the following differential equations

a) \[ \frac{d^2 y}{dx^2} + 4y = x^2 + \sin 2x \]

b) \[
\begin{aligned}
\frac{dx}{dt} + 2x + 3y &= 0 \\
3x + \frac{dy}{dt} + 2y &= 2e^{2t}
\end{aligned}
\]

5  a) Show that the following system of equations is consistent, and find its solution.

\[
\begin{align*}
x + 2y - 2z &= 1 \\
2x - 3y + z &= 0 \\
5x + y - 5z &= 1 \\
3x + 14y - 12z &= 5.
\end{align*}
\]

b) Test for convergence the series

\[ 1 + 3x + 5x^2 + 7x^3 + \cdots \quad (x > 0) \]
6 a) Find the Laplace transform of the function
\[ f(t) = \begin{cases} 
1 & 0 \leq t < 1 \\
t & 1 \leq t < 2 \\
t^2 & 2 \leq t < \infty 
\end{cases} \]

b) If \( L\{\cos at\} = \frac{s}{s^2 + a^2} \) using this result find \( L\{\sin at\} \)

7 a) Expand \( \sin x \cos y \) in powers of \( x \) and \( y \) as far as terms of third degree.

b) For the matrix
\[
A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}
\]
verify that \([\text{Adj}(A)]^T = \text{adj}(A^T)\)

8 a) Is the vector \( \vec{V} = (e^x \sin y) \hat{i} + (e^x \cos y) \hat{j} \) irrotational?

b) Using Laplace transform to solve the differential equation
\[
\frac{d^2x}{dt^2} - 2 \frac{dx}{dt} + x = e^t
\]
with \( x = 2, \frac{dx}{dt} = -1 \) at \( t = 0 \).