BF-5328 Seat No.________

Third Year B. Sc. Examination

May/June - 2014

Mathematics : Paper-X

[Topology]

(Old Course)

Time : 3 Hours] [Total Marks : **105**

1 (a) If $I:(X,d)\to(X,d);\ I(x)=x,\ \forall x\in I$; then I is continuous on X. **7**

(b) If $d_Q : Q\times Q\to R;\ d_Q(a,b)=|a-b|$; then prove that (Q, d_Q) is a subspace of (R,d); where Q be the set of rational numbers. **7**

(c) Give an example to show that the union of closed sets need not closed. **7**

OR

1 (a) If $d: R^n\times R^n\to R;\ d(x,y)=\sqrt{\sum_{i=1}^{n}(x_i-y_i)^2}$; then prove that : (R^n, d) is a metric space; where $x=(x_1, x_2, \ldots\ldots\ldots x_n)$; $y=(y_1, y_2, \ldots\ldots\ldots y_n)\in R^n$. **7**

(b) Show that : **7**

A set F is closed in metric space

iff

F contains all its limit points.

(c) $f:(X,d)\rightarrow(Y,d')$ is continuous **7**

iff

$f^{-1}(O)$ is an open subset of X; where O is an open set of Y.

2 (a) Let $Z=\{1,2,3,......\}$ and **7**

$O_n=\{n, n+1, n+2,.....\}; n\in N$ and let

$T=\{\phi, O_1, O_2,.....O_n.....\}$. Show that : (Z,T) is a topological space.

(b) Prove that : **7**

A subset O of topological space X is open iff O is a neighborhood of each of its points.

(c) Prove that : **7**

A subset A of a topological space X is closed

iff

$A=\overline{A}$

OR

2 (a) Prove that : **7**

$A\cup Bdry(A)=\overline{A}$; where A is a subset of a topological space X.

(b) If $X\neq\phi$ and **7**

$T=\{A\subset X \mid C(A)=\phi \text{ or } C(A)=\text{finite}\}$; then show that : (X,T) is a topological space.

(c) Prove that : **7**

If A be a subset of a topological space X; then Int.(A) is the largest open set contained in A.

3 (a) Prove that : **7**

$f:(X,T)\to(Y,T')$ is continuous

iff

for each subset A of X; $f(\overline{A})\subset\overline{F(A)}$.

(b) Prove that : **7**

Inclusion map $i:Y\to X$ is continuous; where Y is a subspace of a topological space X.

(c) Prove that : **7**

$f:Y\to X$ is continuous

iff

$f':Y\to f(Y)$ is continuous.

OR

3 (a) Let (X,T) be a topological space and let Y be a subset of X. Let **7**

$T'=\{O'\subset Y \mid O'=O\cap Y;\text{ where } O\in T\}$.

Then prove that (Y,T') is a topological space.

(b) A function $f:(X,T)\to(Y,T')$ is continuous **7**

iff

for each open subset O of Y; $f^{-1}(O)$ is an open subset of X.

(c) Let A be a subset of topological space X, Prove that : **7**

A is closed iff $Bdry(A)\subset A$.

4 Attempt any three : **21**

(1) Prove that :

Continuous image of connected set is also connected.

(2) State and prove : Intermediate-value theorem.

(3) State and prove : Fixed-point theorem.

(4) Prove that : The closure of a connected set is connected.

5 Attempt any three : **21**

(1) Prove that : Continuous image of a compact space is compact.

(2) Prove that : If X and Y be compact topological spaces, then $X \times Y$ is also compact.

(3) State and theorem : LINDELÖF'S THEOREM.

(4) Prove that : If $f : X \to Y$ is uniformly continuous; then f is continuous.