* B S - 1 3 6 3 *

BS-1363

Seat No._________

B. Sc. (Sem. IV) Examination

March / April - 2014

CC MATH-402 : Advanced Linear Algebra

Time : 3 Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.

(ii) Figure in the right side indicate the marks of the question.

1 (a) Prove that an n×n matrix A is invertible if and only if the corresponding linear map T of A with respect to the standard basis is non-singular. **8**

(b) Attempt any two : **12**

(i) Obtain the linear transformation associated

with the matrixn $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 0 \end{bmatrix}$ by

taking the standard basis of R^3 and R^4.

(ii) $T : R^3 \to R^2$ and

$T(e_1) = (2,-2), T(e_2) = (1,2), T(e_3) = (0,0)$

$B_1 = \{(1,1,0), (1,0,1), (0,1,1)\}$ and

$B_2 = \{(1,1), (1,-1)\}$ are basis of R^3 and

R^2 respectively.

Find $[T : B_1, B_2]$.

(iii) Find range, null space and rank, nullify

of matrix $A = \begin{bmatrix} 5 & 0 & 1 & 6 \\ 0 & 0 & 3 & 3 \\ 1 & 0 & -1 & 0 \end{bmatrix}$.

2 (a) State and prove triangular inequality. 8

OR

(a) Let V be a vector space which is an inner product space then $|\langle x, y \rangle| = \|x\| \|y\|$ iff x and y are linearly dependent. 8

(b) Attempt any two : **12**

(i) Using Gram-Schmidt process obtain the orthonormal basis from the basis $\{(-1,1,1),(1,-1,1),(1,1,-1)\}$.

(ii) Define an inner product space. In a vector space R^2 for $x=(x_1, x_2)$, $y=(y_1, y_2)$ define $\langle X, Y\rangle = x_1y_1 + x_1y_2 + x_2y_1 - 3x_2y_2$ then show that $\langle X, Y\rangle$ is an inner product in R^2.

(iii) Define a dual basis. Find dual basis for the base $\{(1,-1,3),(0,-1,-1),(0,3,-2)\}$.

3 (a) State and prove Cayley-Hamilton theorem. **8**

OR

(a) Prove that eigen vectors of symmetric linear map corresponding to different eigen values are perpendicular to each other. **8**

(b) Attempt any two : **12**

(i) Using Cayley-Hamilton theorem find the inverse of matrix $A=\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 5 & 3 \end{bmatrix}$.

(ii) Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(iii) Find the minimal polynomial for matrix

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}.$$

4 Attempt any four : **10**

(i) Prove that $(AB)^T = B^T A^T$.

(ii) For a linear map

$T : R^2 \to R^2$ $T(\alpha, \beta) = (2\alpha + \beta, \alpha + 3\beta)$ find T^*.

(iii) State and prove parallelogram law.

(iv) Solve the system of following equation by using Echelon matrix.

$2x + 3y - z = 1,\ x - 3y + 2z = -1,\ x - y + 2z = 2$

(v) Define :

(i) inner product space

(ii) minimal polynomial
